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ABSORPTION OF γ-RAYS

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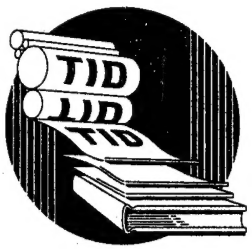

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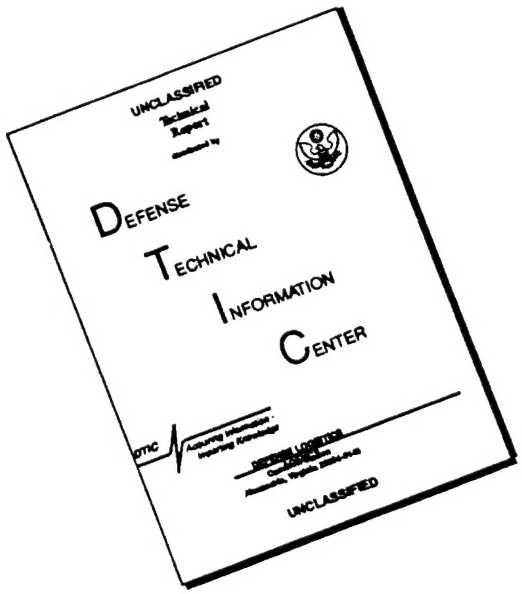

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ABSORPTION OF γ- RAYS

J. L. Powell and W. S. Snyder

In the evaluation of experiments on γ-ray attenuation and in the general problem of shield design it is necessary to have accurate knowledge of absorption coefficients as a function of γ-ray energy and of atomic number. Even a cursory examination of the literature reveals suprisingly large discrepanceis among the values quoted by various authors, differences of five or ten percent of the total coefficient being not uncommon.

An attempt is made here to compile a table of coefficients which is representative of the best experimental data available at the present time, and to give an estimate of the accuracy of the values given. Except for a quite limited range of the variables involved, it has been found that theoretical formulae, as given in standard textbooks, cannot in general be relied upon to give the absorption coefficients with an accuracy of five percent. This seems to be attributable, in most cases, to various approximations which enter the theoretical calculations of cross-sections for photoelectric absorption and for pair production.

Photoelectric Effect

According to the theory of photoelectric absorption, as given by Heitler [1] the photoelectric cross-section is proportional to Z^5, where Z is the atomic number. This result is based upon a calculation which takes account of the K electrons only. Also, the assumption is made that the K electron is adequately described by a hydrogen-like wave function. As a consequence, no account is taken of the effect of the outer shells, even as regards their influence on the K electron. For example, the K absorption limit comes out at Z^2 x 13.5 ev

in this theory, a value which is known from experiment to be too high for heavy elements. It seems possible that the calculated cross-section for lead could be as much as 10% in error due to this approximation. The influence of the nuclear Coulomb field on the photoelectron is discussed by Heitler, and the result of an exact non-relativistic calculation by Strobbe[2] is given in the form of a correction factor which should be applied for energies near the K-absorption limit. This factor is 0.12 at the K-absorption limit, and at an energy of 50 times the absorption limit is 0.66. Even for the element aluminum, this "Coulomb factor" is still appreciably smaller than unity when the γ-ray energy is 1 Mev. However, the non-relativistic calculation of Strobbe is not correct except for energies which are small compared to the rest energy of the electron ($\sim$1/2 Mev).

A relativistic calculation by Sauter[3], in which the influence of the Coulomb field on the photoelectron is neglected (Born approximation), leads to a formula which is adequate to describe the cross-section due to K-electrons for light elements and high γ-ray energy. The restriction to light elements is due to the use of the Born approximation, which is valid only if $Z \ll 137$.

A smooth transition from the non-relativistic region in the neighborhood of the K absorption limit to the extreme relativistic region may be obtained from Sauter's formula by multiplying it by the correction factor of Strobbe. Since the correction is still appreciable at relativistic energy, this procedure is not theoretically justified, but it nevertheless seems to be a reasonable approximation for light elements. At high energy the Compton effect contributes the major part of the total cross-section for light elements so that a relatively large error in the photoelectric cross-section may be tolerated.

For heavy elements, the above approximations are not valid. Exact calculations have been made by Hulme[4] and co-workers at energies up to 2 Mev. The results of these calculations are given in Fig. I, which is taken from reference 4. It is evident from these graphs that the theory based upon the Born approximation, which leads to the "Z^5 law" gives at best an order of magnitude estimate of the actual photoelectric cross-section.

Finally, it should be mentioned that all of the above theoretical work has dealt with the photoelectric effect in the K-shell only. In most cases the effect of the other electronic shells has been estimated from the empirical rule that the K-shell is responsible for about 80% of the total absorption. Some theoretical justification for this rule has been obtained by Hall and Rarita[5], who calculated the cross-section of the L electrons in lead at $\lambda = 4.7$ X.U. with the result $\sigma_L/\sigma_K = .20$.

On account of the complexities discussed above, it is difficult to evaluate theoretical calculations as to the accuracy with which they will give the photoelectric cross-section for a given element at a given γ-ray energy. In general, it is found that the existing theoretical calculations are confirmed by experiment within rather narrow limits, but errors of the order of ten per-cent in the cross-section may be expected in some cases, particularly for heavy elements.

It has been found from experiments [6][7] at x-ray wave lengths that the photoelectric absorption coefficient is well described, for a limited range of Z, by empirical formula

$$\mu \sim Z^p$$

in which p is a rather slowly varying function of the energy. In the neighborhood of the K-absorption limit, $p \approx 4$. This formula is not applicable over the entire

range of Z, but a good approximation can be obtained by dividing the range into two parts and evaluating p for each. In the present work the division was made at Z≈40. This procedure is valuable for purposes of interpolation.

Compton Effect

The contribution to the total absorption coefficient which is due to electron scattering is given by the Klein-Nishina[8] formula. This theoretical formula applies to scattering by free electrons only. However, since the binding energies of atomic electrons are in general quite small compared to the γ-ray energies which are of interest here, one is justified in neglecting the effect of binding.

The Klein-Nishina formula is of a universal nature, since the Compton cross-section is rigorously proportional to Z, so that a single detailed calculation is sufficient to give the Compton cross-section for all elements.

Pair Production

At energies above 1 Mev the production of electron-positron pairs in the Coulomb field of the nucleus makes an important contribution to the γ-ray absorption. A thorough theoretical discussion of this effect has been given by Bethe and Heitler[9], who calculated the pair production cross-section using the Born approximation, in which it is assumed that the pair electrons may be treated as free particles. This approximation is expected to fail for heavy elements, and it is generally assumed to be accountable for discrepancies which have recently been found in comparing experimental to theoretical cross-sections at energies above 10 Mev.[10][11][12]

For energies well above the threshold, it is necessary to include the effect of screening of the nuclear Coulomb field by the atomic electrons.

The rather complicated formulae of Bethe and Heitler have been approximated in a simple manner by Hough[13].

For small values of Z, for which the Born approximation is valid, the theoretical formulae have been well confirmed by experiment, and accurate cross-sections may be obtained from them with a moderate amount of calculation. Reliable theoretical formulae do not exist for large Z values, and it is necessary to rely practically entirely on experimental results in this region. The Bethe-Heitler formula gives a cross-section for lead which is about 10% too large for γ-ray energies in the range 10 to 100 Mev.

In addition to the principal effect of pair production in the nuclear Coulomb field, there is a small contribution due to pair production in the fields of atomic electrons, which is approximately proportional to Z.[14][15][16] Provisionally, this may be included, at least as to order of magnitude, by replacing the Z^2 of the Bethe-Heitler formula by $Z(Z+1)$. More accurate estimates of this effect are given in the references cited, but for the purposes of interpolation in Z, the above prescription is adequate.

Experimental Values

In Table I are given the experimental absorption coefficients which are the basis of the more complete Table II.

The data of Cuykendall and Jones are included as being the most complete and representative of modern experiments in the X-ray region. Much of the data of these experimenters, which relates to the low energy region in the neighborhood of the absorption limit, has been omitted, since it is of minor importance for shielding applications. The data of Cowan are the results of extensive experiments using radioactive souces of γ-rays and a Geiger-Muller tube as detector. The energy range is .32 to 2.3 Mev. G. D. Adams' work was done with the Illinois betatron using threshold detectors which were sensitive to a small portion of

the spectrum at the high energy limit. Copper, iron and carbon detectors were used, giving average energies at 11.04, 13.73 and 19.1 Mev. The 17.6 Mev γ-rays from the reaction $Li^7(p,\gamma)$ Be^8 were used by R. L. Walker in very careful measurements of the cross-sections for C, Al, Cu, Sn, and Pb. The detecting device in these experiments was a magnetic pair spectrometer which made it possible to eliminate a background of lower energy γ-rays. The data of Lawson at 88 Mev were obtained using the 88 Mev γ-rays from the G. E. betatron and a pair spectrometer as detector.

Both Cowan and Adams report their observed absorption coefficients in units of cm^{-1}, and it was necessary to convert them to the units used here by dividing by the density of the material. The densities assumed are

Al	2.7 g/cm^3
Fe	7.85
Cu	8.89
Sn	7.18
Pb	11.1

A small error may have been introduced in this conversion of the data, but it is not likely that it is larger than 2%.

Adjustment of Theoretical Curves

The data of Table II and the curves in Supplement I were obtained by adjustment of theoretical curves to fit the experimental data of Table I. The formulae for photoelectric, Compton and pair production cross-sections as given by Heitler, were used to compute "theoretical" cross-sections for the various elements. The relativistic (Sauter) formula for the photoelectric cross-section was used in conjunction with the Coulomb correction factor as discussed above. The Compton cross-sections were computed exactly from the Klein-Nishina formula except that no correction was made, even at the lowest energies, for electron binding. The

Bethe-Heitler pair production formula was used, including the shielding effect at high energies. The latter was interpolated from curves given by Heitler (p. 201). The effect of pair production in the electronic fields was included by replacing the Z^2 of the Bethe-Heitler formula by $Z(Z+1)$.

It is clear that these calculations are not sufficiently accurate to provide a comparison between experiment and theory. Nevertheless it was found that for the light elements, e.g., Al, the values given by the formulae were in close agreement with experimental results. For larger values of Z the agreement is progressively worse, and in the case of lead it was found that the calculated values were larger than the experimental by about 10% over the range 1 to 100 Mev. In the photoelectric region, the agreement for lead is much better than at the higher energies. This is undoubtedly fortuitous, since the intermediate values of Z (e.g., Sn), the formulae give coefficients which are appreciably smaller than the experimental values.

In the construction of the final curves, the elements Al, Cu, Sn and Pb were selected as reference elements, since most experimental information is available for these. The experimental points were plotted and compared to the calculated curves, which were then adjusted graphically to fit the experimental data as closely as possible. In this way, the formulae were made to serve as a means of interpolation for energy ranges which were not covered by experiment. In the case of Al, practically no significant adjustment was necessary, and for Cu and Sn, only small corrections were required, principally at low energy.

The values for the elements Fe, Ag, Ta and U were interpolated from the results for the reference elements by first subtracting the calculated Compton coefficient (Supplement 2) and then adjusting the remaining photoelectric and pair production parts by the semi-empirical methods discussed above.

For X-ray energies the exponent p was obtained from the work of Cuykendall[6] and Jones[7]. This is given explicitly by Jones for $Z > 40$, and may be computed easily from the data of Cuykendall for $Z < 40$ (see Table 3). The variation of p is not great in the energy range under consideration, and reasonably good extrapolations can be made, which compare well with the results of Hulme et al, at higher energies. Hulme's results apply to the range .4 to 2 Mev and can be obtained directly from the graph of Fig. 1.

The pair production cross-sections were in all cases assumed to be proportional to $Z(Z+1)$, which is apparently quite a good approximation for small ranges of Z, as can be seen, for example, by comparison of the Fe curve, which was based on Cu, to the experimental results of Adams at 11, 14 and 19 Mev.

In all cases, the adjusted curves give the experimentally observed coefficient to within 3% of the total. In those regions where experimental data are not available, it is possible that the interpolated curves are less accurate, but it seems unlikely that the error should exceed 5%.

It would be of great value to have further experimental data for uranium, since the values given here for that element have been extrapolated from lead over a relatively large interval in Z. Also, the intermediate range of energies in the neighborhood of the minimum absorption for lead have not been adequately investigated experimentally.

In Supplement 2 are plotted absorption coefficients for elements lighter than Al. These are the results of calculations from the formulae described above. Practically no experimental data are available for these elements, but very good check obtained for aluminum justifies the belief that the formulae are correct in this region.

Supplement 3 and Supplement 4 give the mass absorption coefficients minus Compton scattering, $\frac{\mu - \sigma_s}{\rho}$.

Table I. Absorption Coefficients - cm^2/g (Experimental)

Mev	$\rho = 2.7\ g/cm^3$ Al(13)	$\rho = 7.85$ Fe(26)	$\rho = 8.89\ g/cm^3$ Cu(29)	Ag(47)	$\rho = 7.18\ g/cm^3$ Sn(50)	Ta(73)	$\rho = 11.1\ g/cm^3$ Pb(82)	U(92)
.0591	.279							
.0671	.240							
.0772	.213		.830					
.0883	.191		.606					
.0950	.176		.513					
.0988	.175							
.103	.168		.434					
.107	.162							
.112	.158		.371					
.123	.150 (1)		.306 (1)	.900				
.130	.147							
.137	.144		.259	.715	.800			
.145	.142							
.155	.138		.215	.544	.607			
.165	.135			(2)	(2)			
.176	.131		.182	.400	.430			
.206	.121		.160	.283	.314	.689	.900	
.225	.117					(2)		
.247	.113		.129	.229	.246	.458	.600 (2)	
.309							.394	
.32	.108		.1053		.146			
.65	.0722 (3)		.0694		.0726		.1135	
1.11	.0574		.0538 (3)		.0519 (3)		.0651	
1.72			.0418		.0421		.0480 (3)	
2.30					.0371		.0426	
11.04	.0224	.0295	.0311				.0517	
13.73	.0221 (4)	.0306 (4)	.0321 (4)				.0568 (4)	
19.10	.0223	.0334	.0346				.0632	
17.6	.0218 (6)		.0344 (6)		.0455 (6)		.0597 (6)	
88	.0252 (5)		.0471 (5)		.0665 (5)		.0909 (5)	.0973(5)

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(2) M. T. Jones, Phys. Rev. 50, 110 (1936)
(3) Clyde L. Cowan, Phys. Rev. 74, 1841 (1948)
(4) G. D. Adams, Phys. Rev. 74, 1707 (1948)
(5) J. L. Lawson, Phys. Rev. 75, 433 (1949)
(6) R. L. Walker, Phys. Rev. 76, 527 (1949)

Table II. Absorption Coefficients (cm^2/g) (Adjusted)

Mev	Al(13)	Fe(26)	Cu(29)	Ag(47)	Sn(50)	Ta(73)	Pb(82)	U(92)
.0313	1.00							
.04	.555							
.05	.355							
.07	.225	.820						
.1	.168	.384	.463					
.15	.136		.225		.643			
.20	.123	.149	.158	.313	.342	.742	.97	1.235
.30	.104		.110		.162		.415	
.40	.092	.092	.091	.110	.1125	.193	.237	.298
.50	.0845		.079		.0905		.163	
.70	.073	.0697	.067	.0700	.069	.0908	.104	.124
1.0	.0613	.0590	.0565	.0565	.055	.0668	.073	.0838
1.5	.0495		.0455		.0445		.0527	
2.0	.0425	.0405	.0395	.0404	.0393	.0409	.0449	.0470
3.0	.0345		.035		.036		.0411	
4.0	.0311	.0331	.0333	.0361	.0356	.0390	.0412	.0442
7.0	.0252	.0302	.031	.0368	.0368	.0422	.0450	.0483
10.0	.0230	.0304	.0316	.0390	.0392	.0462	.0495	.0536
20.0	.0214	.0331	.0352	.0474	.0483	.0601	.0645	.0706
40.0	.0229	.0384	.0413	.0572	.0585	.0738	.0800	.0890
70.0	.0242	.0421	.0456	.0634	.0650	.0812	.0885	.0970
100.0	.0249	.0431	.0468	.0662	.0680	.0830	.0903	.0993
.072			1.00					
5.0			.032				.0425	
15.0			.0335				.0586	
.197							1.0	
.25							.600	
30.0							.0750	
50.0							.084	

TABLE 3

Exponent for Photoelectric Effect
Estimated from Reference 6 ($Z<40$)

Mev	p
.06	4.14
.08	3.96
.10	3.82
.12	3.72
.14	3.66
.16	3.61
.18	3.58
.20	3.56

Exponent for Photoelectric Effect ($Z>40$)
Reference 7

λ(XU)	Mev	p
140	.0882	3.51
130	.095	3.64
120	.103	3.62
110	.112	3.68
100	.123	3.69
90	.137	3.69
80	.155	3.73
70	.176	3.84
60	.206	3.88
50	.247	3.84

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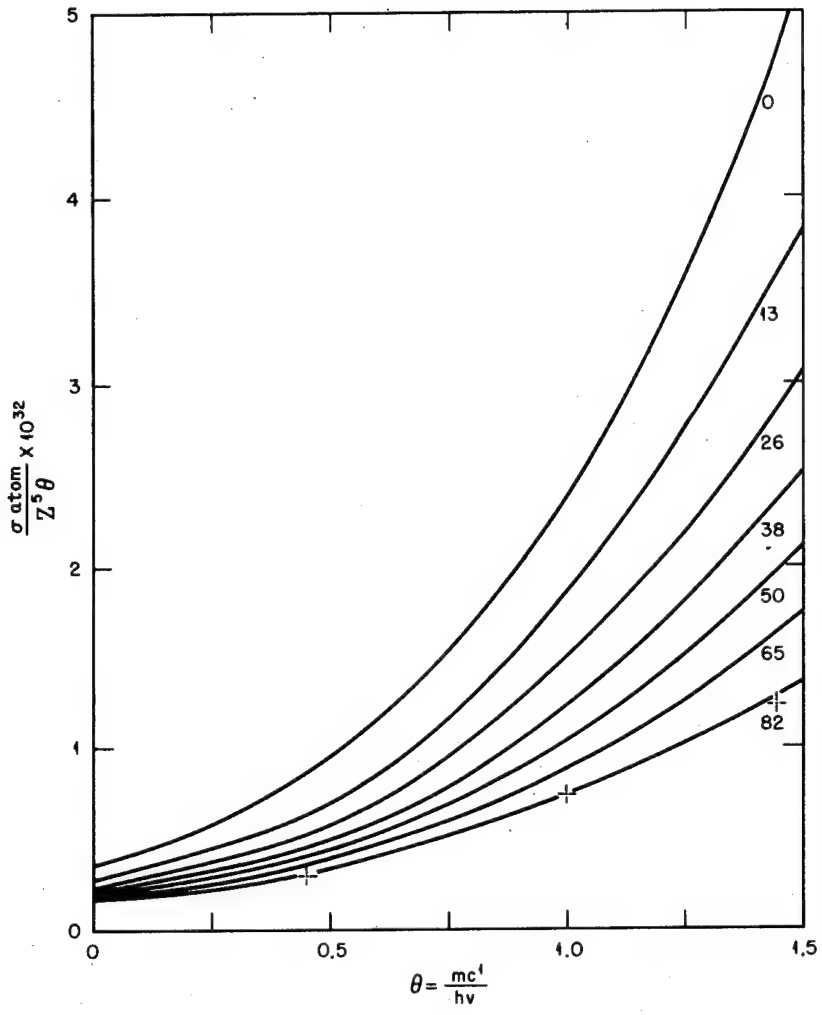
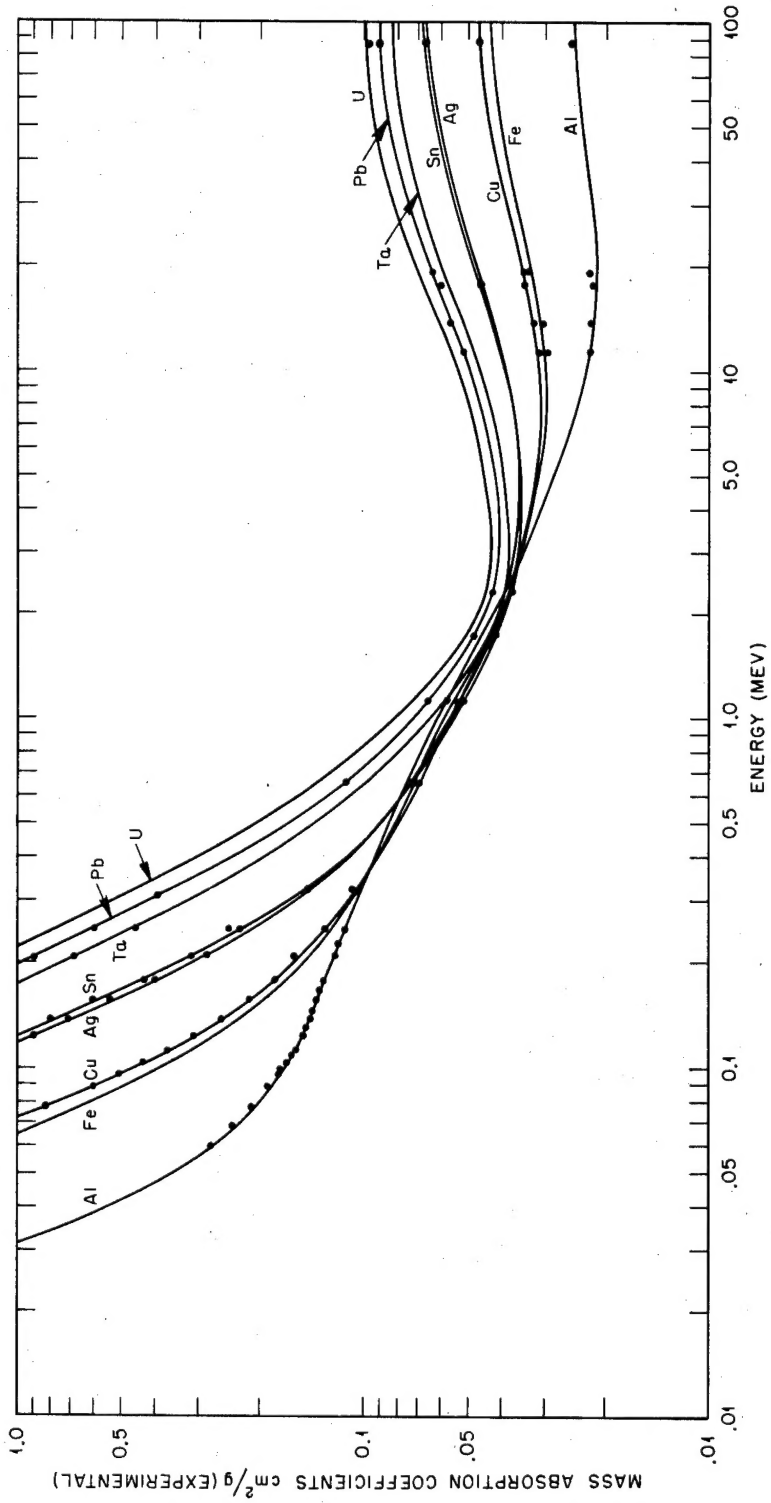
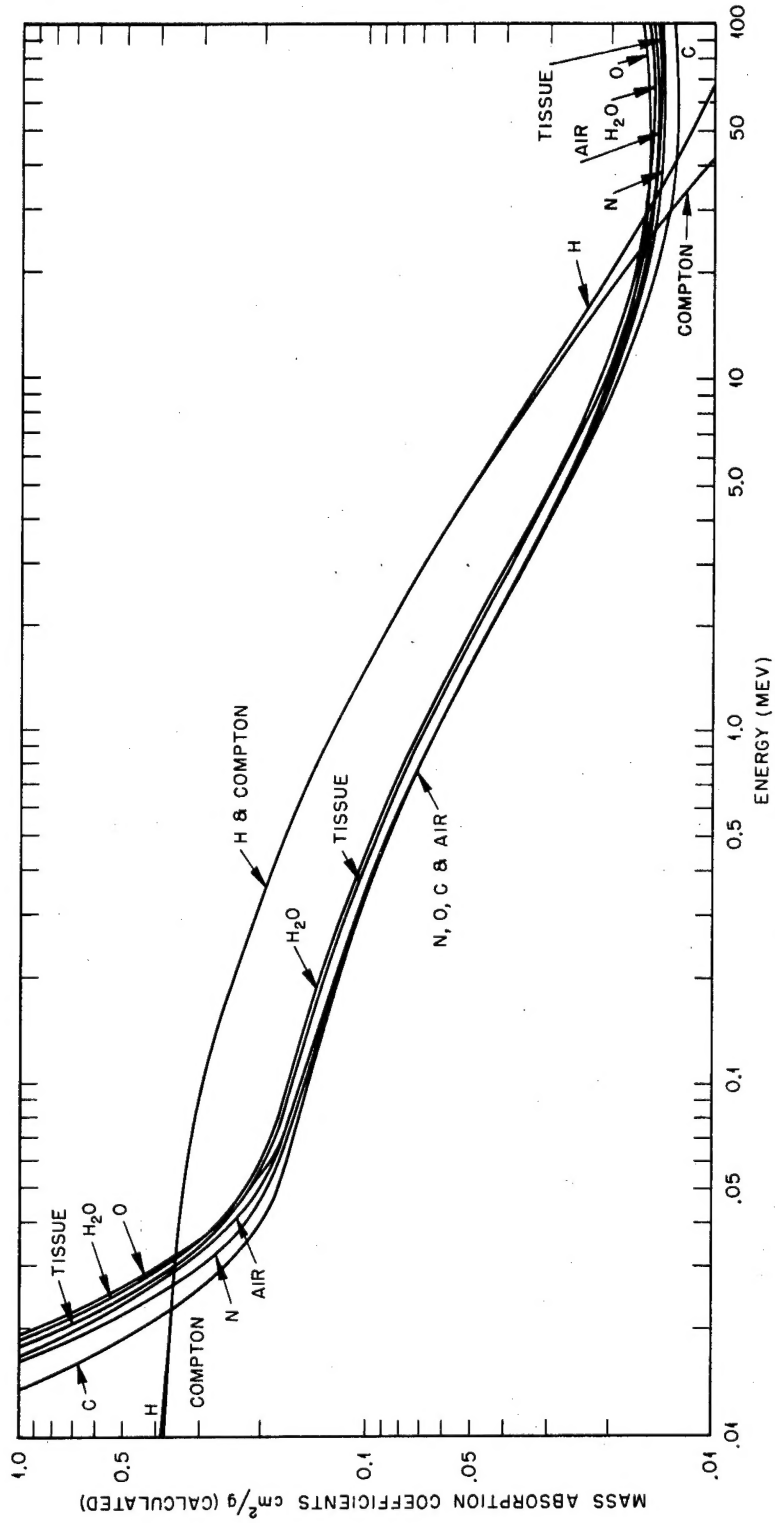

5
4
3
2
1
0
0
0.5
1.0
1.5
0
13
26
38
50
65
82
$\frac{\sigma\,atom}{Z^5\theta} \times 10^{32}$
$\theta = \frac{mc^2}{h\nu}$

Fig. 1



Supplement 1



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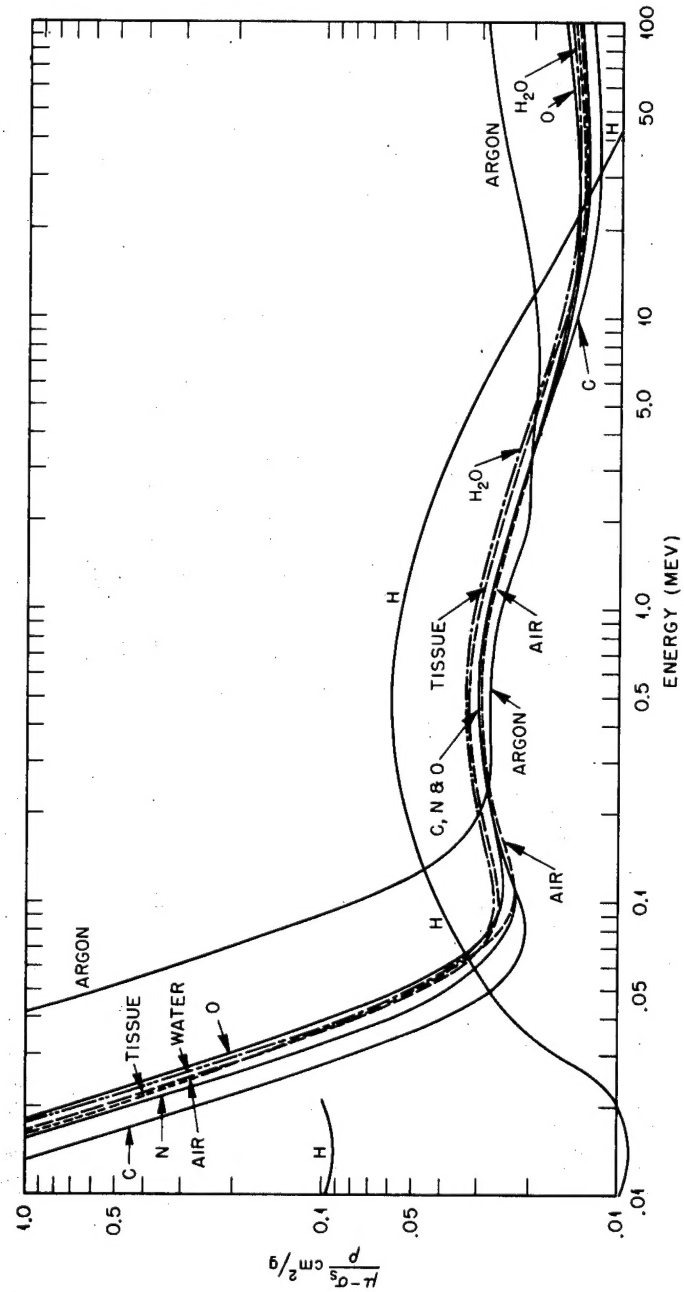

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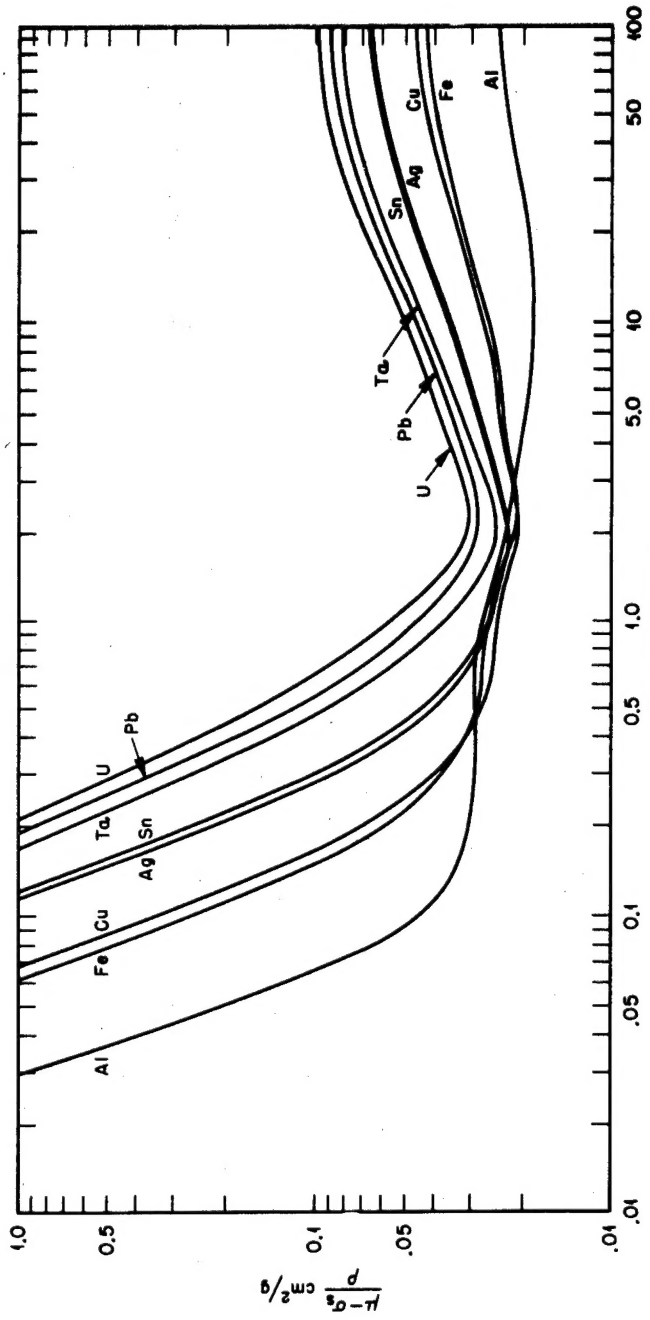

Supplement 4

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